Shared Risk Link Groups of Disaster Failures

Balázs Vass*,
*Department of Operations Research, Eötvös University, Budapest, Hungary, vbalazs@cs.elte.hu
MTA-BME Lendület Future Internet Research Group, Budapest University of Technology and Economics (BME)

***Abstract*—Current backbone networks are designed to protect a certain pre-defined list of failures, called Shared Risk Link Groups (SRLG). A common assumption is to ignore the failures not part of an SRLG as they assume to be extremely rare events; therefore, the list of SRLGs must be defined very carefully. The list of SRLGs is typically composed of every single link or node failure. It has been observed that some type of failure events manifested at multiple locations of the network, which are physically close to each other. Such failure events are called regional failures, and are often caused by a natural disaster. The number of possible regional failures can be very large, thus simply listing them as SRLGs is not viable solution. In this study we focus only to every regional failure with a shape of disk that do not contain network nodes. We provide a fast systematic approach for generating a list of SRLGs the protection of which is essential to increasing the observed reliability of the networks. According to some practical assumptions this list is very short with $O(|V|)$ SRLGs in total, and can be computed very fast, in $O(|V|\log|V|)$.**

I. Introduction

Current backbone networks are built to protect a certain list of failures. Each of these failures (or termed failure states) are called *Shared Risk Link Groups* (SRLG), which is a set of links that is expected to fail simultaneously. The network is designed to be able to automatically reconfigure in case of a single SRLG failure, such that every connection further operates after a very short interruption. In practice it means the connections are reconfigured to by-pass the failed set of nodes and links. The list of SRLGs must be defined very carefully, because not getting prepared for one likely simultaneous failure event the observed reliability of the network significantly degrades.

On one extreme is listing every *single link or node failure* as an SRLG. Often there is a known risk of a simultaneous multiple failure that can be added as an SRLG, for example if two links between different pair of nodes traverse the same bridge, etc. On the other hand, we have witnessed serious network outages because of a failure event that takes down almost every equipment in a physical region as a result of a disaster, such as weapons of mass destruction attacks, earthquakes, hurricanes, tsunamis, tornadoes, etc. For example the 7.1-magnitude earthquake in Taiwan in Dec. 2006 caused simultaneous failures of six submarine links between Asia and North America and hurricane Sandy in 2012 cased a power outage that silenced 46% of the network in the New York area. These type of failures are called **regional failures**. It is still a challenging open problem how to prepare a network to protect such failure events, as their location and size is not known at planning stage. In the study we propose a solution to this problem with a technique that can significantly reduce the number of possible failure states that should be added as an SRLG to cover all regional failures that does not affect any node.

In practice, regional failures can have any location, size and shape. It is a common best practice to fix the size or shape of regional failures, for example as cycles with given size (also called disk) [1]. Another approach to analyse the network vulnerability against regional failures is using probabilistic failure models, where each link in the SRLG has some probability to fail [2].

In this study we show an efficient way to generate the SRLGs of **single regional failures** which erase the network elements in a circular area and do not affect nodes. Based on the model and assumptions described in Section II, we have shown that with these assumption the number of SRLGs is small, $O(|V|)$, in typical backbone network topology, and can be at most $O(|E||V|)$ in an artificial worst case scenario. We propose a systematic approach based on computational geometric tools that can generate the list of SRLGs in $O(|V|\log|V|)$ steps on typical networks.

We believe this result is a step towards filling the gap between the conventional SRLG based pre-planned protection and regional failures.

II. Model, Assumptions and Results

We model the network as an undirected geometric graph $G(V,E)$ with $n=|V|$ nodes and $m=|E|$ edges, we assume $n\geq 3$. The nodes of the graph are embedded as points in the Euclidean plane, and the edges are embedded as line segments. The position of node v is denoted by (v_x, v_z). A *disk failure* $k(x,y,r)$ is a circle with a centre point (x,y) and radius r. The failure is modelled as every interior node and edge with interior part is erased from the graph.

Let K_0 denote the set of possible disk failures that do not have any node of V in interior. We call K_0 as link failures. Clearly, $|K_0|$ is infinite. Recall that our task is to generate a set of SRLGs, thus instead of a disk failure $k\in K_0$ we are rather interested in the set of links denoted by m_k, interior to k. Let $k_1,k_2\in K_0$ be two disk failures such that $m_{k_1}\subset m_{k_2}$. We assume if the network can survive failure of k_2 it can survive k_1 as well. Thus we focus on computing of the set $\mathcal{M}_0$ of inclusion-wise maximal SRLGs caused by elements of K_0.

Observation 1. *For any $k_1\in K_0$ there exists a $k_2\in K_0$ such that $k_1\subseteq k_2$ and k_2 has at least 2 points of V on its boundary.*

Let $K_0^{u,v}$ be the set of disks from K_0 which have both nodes u and v on the boundary. According to Obs. 1, it is enough to determine the maximal SRLGs caused by disks in $\cup_{u,v \in V} K_0^{u,v}$.

Let denote $D_0 = (E_0, V)$ the Delaunay triangulation [3] on the set of nodes. An important observation is the following.

Observation 2. *$K_0^{u,v}$ is non-empty iff $\{u, v\}$ is an edge of the $D_0 = (E_0, V)$ Delaunay triangulation.*

Let denote $\mathcal{M}_0^{u,v}$ the set of maximal elements among SRLGs caused by the elements of $K_0^{u,v}$. Clearly, sets $\mathcal{M}_0^{u,v}$ and $\mathcal{M}_0^{w,z}$ are not necessarily disjoint and their elements are not necessarily globally exclusion-wise maximal.

This gives us the idea to solve the problem according to the following plan. First generate the $D_0 = (E_0, V)$ Delaunay triangulation. After that for every $\{u, v\} \in E_0$ generate sets $\mathcal{M}_0^{u,v}$. Finally compute $\mathcal{M}_0$ by gathering the globally maximal elements of sets $\mathcal{M}_0^{u,v}$.

Use the parameters θ_0 and τ_0 for the maximum number of edges crossing the circumcircle of a Delaunay triangle, and for the maximum number of circumcircles of Delaunay triangles crossed by an edge, respectively.

Using the previous plan and the specific properties of the Delaunay triangulation we proved the next theorem.

Theorem 1. *$\mathcal{M}_0$ can be computed in $O(n(\log n + \theta_0^3 \tau_0))$ time, and has $O(n\theta_0)$ elements, each of them consisting of $O(\theta_0)$ edges.*

Corollary 1. *Assuming θ_0 is upper bounded by a constant and τ_0 is $O(\log n)$, $\mathcal{M}_0$ can be computed in $O(n \log n)$ time, and the total length of it is $O(n)$.*

A graph family may have $O(n^3)$ single regional failures and we managed to give an artificial graph family, which has $\Theta(n^3)$ of them. However, we are convinced that θ_0 is small in case of typical backbone networks and there exists a small constant c that it never exceeds and thus $|\mathcal{M}_0| \leq cn$.

III. The Algorithm

Since the Delaunay triangulation itself has an optimal $O(n \log n)$ calculation time, the best complexity our approach can reach is $O(n \log n)$.

In order to achieve the $O(n \log n)$ typical time complexity on the one hand we needed to prove that m is not large. Fortunately:

Observation 3. *The number of edges is $O(n\theta_0)$.*

On the other hand when calculating the edge sets covered by the circumcircles of Delaunay triangles instead of intersecting an edge with all circumcircles we managed to intersect it only with "close" circumcircles due to Lemma 1; and when eliminating the redundant elements from the lists $\mathcal{M}_0^{u,v}$, we needed to compare a list with only $O(\tau_0)$ other lists.

Lemma 1. *The set T_e of the Delaunay triangles having circumcircles covering edge e is connected in the sense that from every element of T_e one can reach every element of T_e through triangles having common edge.*

Algorithm 1: Generating the SRLGs of the single regional failures

Data: $G = (V, E)$
Result: The set $\mathcal{M}_0$ of SRLGs of maximal single regional failures.

1 $D_0 = (V, E_0) \leftarrow$DELAUNAY$(V)$;
2 **for** $\{u, v\} \in E_0$ **do**
3   some preparation
4 $E_{u,v}^i \leftarrow$ GETEDGESETS$(D_0 = (V, E_0), E)$;
5 **for** $\{u, v\} \in E_0$ **do**
6   GENERATE $\mathcal{M}_0^{u,v}$
7 $\mathcal{M}_0 \leftarrow$ ELIMINATEREDUNTANTS$(\mathcal{M}_0^{u,v}, \forall \{u, v\} \in E_0)$;
return $\mathcal{M}_0$

Algorithm 1 is a sketch of the main ideas. At line 1 the Delaunay triangulation is computed in $O(n \log n)$ time. In lines 2 and 3 we do some preparation in constant time for every $\{u, v\}$ Delaunay edge, such as determining the Delaunay triangles $t_{u,v}^1$ and $t_{u,v}^2$ having $[u, v]$ as their edge (if $\{u, v\}$ is on the convex hull of V, we treat it specially). Let $C_{u,v}^i$ be the circumcircle of $t_{u,v}^i$, and $E_{u,v}^i$ be the set of edges crossing $C_{u,v}^i$ (see Fig. 1).

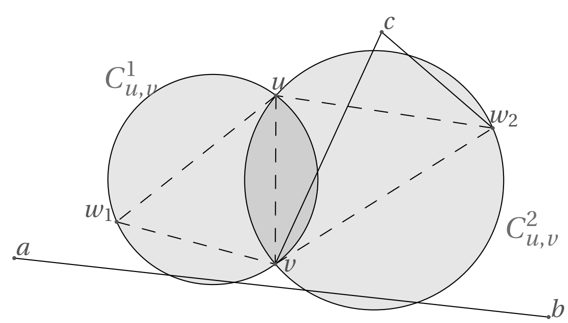

Fig. 1: Example of a Delaunay-edge $\{u, v\}$ with $C_{u,v}^1$ and $C_{u,v}^2$. Here $E_{u,v}^1 = \{\{a, b\}, \{c, v\}\}$, $E_{u,v}^2 = \{\{a, b\}, \{c, v\}, \{c, w_2\}\}$.

In line 4 we calculate $E_{u,v}^i$ simultaneously for all $\{u, v\} \in E_0$ in $O(n\theta_0^2)$ time. In lines 5 and 6 we generate sets $\mathcal{M}_0^{u,v}$, each in $O(\theta_0^2)$ time. Finally, in line 7 $\mathcal{M}_0$ is calculated from sets $\mathcal{M}_0^{u,v}$ in $O(n\theta_0^3\tau_0)$ time.

According to these results we can derive Theorem 1.

References

[1] S. Neumayer, A. Efrat, and E. Modiano, “Geographic max-flow and min-cut under a circular disk failure model,” *Computer Networks*, vol. 77, pp. 117–127, 2015.

[2] S. Neumayer, G. Zussman, R. Cohen, and E. Modiano, “Assessing the vulnerability of the fiber infrastructure to disasters,” *Networking, IEEE/ACM Transactions on*, vol. 19, no. 6, pp. 1610–1623, 2011.

[3] F. Aurenhammer, “Voronoi diagramsÑa survey of a fundamental geometric data structure,” *ACM Computing Surveys (CSUR)*, vol. 23, no. 3, pp. 345–405, 1991.