

Abstract

Large scale natural disasters can have a profound effect on the telecommunication services in the affected geographical area. Hence, it is important to develop routing approaches that may help in circumventing damaged regional areas of a network. This prompted the development of geographically diverse routing schemes and also of disaster-risk aware routing schemes. A minimum-cost geodiverse routing, where a minimum geographical distance value D is imposed between any intermediate element of one path and any element of the other path is presented. Next the problem of the calculation of a D -geodiverse routing solution which ensures a certain level of availability is tackled. An algorithm is described that either obtains a solution to that problem or the most available path pair satisfying the desired geographical distance value D – this can be useful for the specification of availability levels in Service Level Agreements. Finally, a case study is presented, in an optical network, to determine the cost increase in terminal equipment (transponders) of approaches to ensure a much larger separation of the paths (of the selected path pair), with respect to minimal length link-disjoint routing.

1 An Introduction to Geographically Diverse Routing

The occurrence of large-scale disasters such as hurricanes, tsunamis, or earthquakes may cause a number of failures with a profound effect in the telecommunication services in a

*This chapter is based on work from COST Action CA15127 (“Resilient communication services protecting end-user applications from disaster-based failures – RECODIS”) supported by COST (European Cooperation in Science and Technology). This work is funded by ERDF Funds through the Centre’s Regional Operational Program and by National Funds through the FCT – Fundação para a Ciência e a Tecnologia, I.P. under the project CENTRO-01-0145-FEDER-029312. This work was also partially supported by FCT under projects UID/EEA/50008/2019 and UIDB/00308/2020. D. Santos was supported by the FCT post-doc grant SFRH/BPD/111503/2015 until November 2018. This work was partially supported by the High Speed Networks Laboratory (HSNLab). Projects no. 123957, 129589, 124171 and 128062 have been implemented with the support provided from the National Research, Development and Innovation Fund of Hungary, financed under the FK_17, KH_18, K_17 and K_18 funding schemes respectively. The work was also supported by the BME-Artificial Intelligence FIKP grant of EMMI (BME FIKP-MI/SC).

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certain geographical area. The lack of communications may have a significant impact in relief operations, as well as in resuming day-to-day activities in the affected areas. In this context, it is important to understand the geographical challenges affecting the connectivity of the network and to provide routing strategies that may help in circumventing damaged areas of the networks. A summary of the papers referenced in this introduction is in Table 1.

Table 1: Summary of papers on geographically diverse routing

Path geodiverse problems	Single region faults region-disjoint paths (or D -geodiverse paths)	region (disaster area) divided in zones
[1]: geographical routing protocol GeoDivRP	[2]: problem definition (only nodes can fail)	[3]: heuristic for selection of disaster resistant routes passing in different zones
[4, 5]: proposal of heuristics iWPSP, MLW	[6]: problem definition [7]: exact problem resolution	
[8]: extension of GeoDivRP, to account for the trade-off between delay and traffic skew on paths	[9]: network augmentation; heuristic	
[11]: extension of GeoDivRP, including critical region identification	[10]: heuristic for GAP selection; new metric for evaluating geographical separation of paths [12, 13]: heuristic based on k -shortest paths	
Multiple region faults		
[14]: heuristics for (i) max number of region-disjoint paths; (ii) min region cut		
[6]: polynomial algorithms for (i) max number of D -geodiverse paths; (ii) min number of disconnecting disks with radius $D/2$		

The path geodiverse problem (PGD) focuses on finding geographically diverse paths for a given node pair. Different metrics may be used to assess the degree of geodiversity between different paths. In [1], the cTGGD (compensated Total Geographical Graph Diversity) value is used as a global graph resilience metric to represent the geographical path diversity of a given topology. An optimal algorithm, GeoDivRP, is proposed for solving the PGD. A follow-up to this work is [5] and its extension is [4], where two heuristics, iWPSP (iterative WayPoint Shortest Path) and MLW (Modified Link Weight), are proposed to reduce the complexity of the optimal routing algorithm. The routing performance

achieved with GeoDivRP may be improved by including information on critical regions, whose identification may be accomplished as proposed in [11]. In [8], a further extension to GeoDivRP is put forward. A mathematical formulation of the GeoDivRP multicommodity linear optimisation problem is presented, and its resolution allows for the optimal traffic allocation information on the multiple paths for all the source-destination node pairs or commodities.

The information on critical regions may be used in the definition of the so-called region-disjoint paths. According to [2], paths are region-disjoint if they cannot be simultaneously affected by a single regional failure of diameter D , assuming that failure does not include the source or destination node. Each intermediate node and each link (without the source or destination node as extreme nodes) in a path has to be at a distance greater than or equal to a threshold D from every intermediate node and link (without the source or destination node as extreme nodes) in the other paths. Region-disjoint paths defined in this way may also be termed as D -geodiverse paths. When dealing with this problem, it is assumed that the geographical proximity between elements (nodes and links) in different paths is known. For specifics on the definition of D -geodiverse paths, see Sect. 4.2, where the work in [7] is discussed in detail.

The region-disjoint paths problem is studied in different works – for more details, see [2] and references therein. A heuristic approach is also proposed in [10] to deal with geographically correlated failures. Given an active path (AP), the algorithm tries to find a backup path (BP) as geographically distant as possible to the AP, while satisfying constraints related to delay and hop count. This is accomplished by an iterative procedure, where the weight of an edge is increased if the distance of that edge to the AP is lower than the predefined distance threshold D . The quality of the solution path pair is evaluated by a measure, the Proximity Factor, which gives a degree of the geographical correlation between the two paths.

Multiple spatial-based simultaneous link failures are considered in [12], where the authors propose a routing method that starts by calculating a set of k shortest paths between two nodes. Afterwards, a heuristic algorithm is used to try and find the link-disjoint path pair that maximizes the minimal distance between the two paths. The authors show that their algorithm allows for a high survivability in a situation of simultaneous link failures in the same region. The same authors propose a variation of this routing algorithm in [13], also aiming at finding the link-disjoint path pair (among a set of k shortest paths) with maximized minimal distance between the two paths, but also guaranteeing that the AP is

the shortest one and the paths satisfy a constraint of minimal distance D between them. In these papers, the authors present details on calculating the minimum spatial distance for a link-disjoint path pair.

In [3], a routing algorithm taking into account the extension of a disaster area is proposed. The disaster area is divided in smaller areas, the upper, middle, and lower parts of the disaster area. The routes going through the upper and lower parts of a disaster area are considered disaster-resistant routes, and the routes traversing the middle part have the shortest total length (minimum-cost routes). Similar routes, which go through the same structures (e.g., pipelines) should be avoided, as they are not disaster-resistant. The authors simulate their model by considering data of an actual disaster area (the area affected by the 2011 Japan earthquake and subsequent tsunami) and manage to calculate adequate candidate routes.

The above mentioned papers deal with single region faults. In [14], the problem of multiple region faults is addressed, i.e, it is assumed that faults may happen simultaneously in different regions. The authors propose heuristic resolution approaches to deal with two different problems: (i) finding the maximum number of region-disjoint paths between a pair of nodes s and d ; (ii) finding the minimum region cut, i.e., finding the minimum number of regions where the occurrence of failures will lead to s and d being disconnected. Papers [6, 15] deal with these latter problems precisely in polynomial time.

This chapter is organized as follows. A description of disaster-risk aware schemes provisioning of communication paths is presented in Sect. 2. Then, in Sect. 4, a description of minimum cost geodiverse routing is given in Sect. 4.2, followed by the introduction of an algorithm for geodiverse disjoint routing with availability constraints in Sect. 4.3. A study on the trade-off between link disjoint routing, geodiverse routing and Shared Risk Link Group (SRLG)-disjoint routing with geodiverse constraints in an optical network is presented in Sect. 4.5. The chapter ends with some final remarks in Sect. 5.

2 Disaster-risk Aware Schemes of Provisioning of Communication Paths

To reduce the impact of disasters on connections, a knowledge on vulnerable regions (e.g., based on statistical information about past disaster events occurred in given areas) is necessary. Such data can be next used to determine the paths for connections in a given network in a way to omit regions of a high probability of disaster occurrence. This idea has

been utilized in [16], where the respective analysis of the risk of traversing via vulnerable regions has been defined and followed by the introduction of an optimization model to establish connections with improved resistance to disasters.

In particular, the risk analysis is presented in [16] in terms of penalty to be paid by a network operator to the customers if a post-disaster service downtime exceeds an allowed downtime specified in the Service Level Agreement (SLA), as given in formula (1):

$$P_n = \sum_{t \in T} c_t Z_t^n (h_r - h_{adt}^t) \quad (1)$$

where:

- c_t is the cost of a failure of connection t (from set of connections T);
- Z_t^n is a binary variable equal to 1 if connection t is affected by disaster n (from disaster set N), 0, otherwise;
- h_r is the average time to recover from a disaster;
- h_{adt}^t is the SLA-regulated allowed service downtime.

This penalty is commonly defined per unit time [17]. As the time h_r of post-disaster recovery is often long (expressed even in terms of weeks) while h_{adt}^t is in SLAs often shorter than one hour (as the 0.9999 availability in a year denotes roughly one hour), which means that $h_r \gg h_{adt}^t$, the post-disaster penalty cost is often dominated by h_r [17]. Such a risk assessment framework can be next used by the traffic engineering scheme to establish connections with a reduced risk of becoming affected by a disaster and with a decreased penalty for the provider.

The optimization model proposed in [16] is to minimize the risk defined in formula (2):

$$R = \sum_{n \in N} \left(\sum_{t \in T} c_t Z_t^n \right) p_d^n p_n \quad (2)$$

where p_d^n and p_n additionally mean the probability of damage implied by disaster n and the probability of disaster n , accordingly, and subject to constraints on: affection of both primary and backup paths by a disaster; on nodal disjointedness of the primary and backup paths for the 1+1 protection model; on flow conservation; and on total link capacity – as presented in [16].

The objective function (2) designed to minimize the risk expressed in terms of the total expected penalty, has, however, a disadvantage of establishing relatively long backup paths. To mitigate this issue, an additional element A defined by formula (3) is proposed in [16] to be summed up to formula (2).

$$A = \epsilon \sum_{t \in T} \sum_{(i,j) \in E} R_{ij}^t \quad (3)$$

where R_{ij}^t is a binary variable equal to 1 if connection t is routed via link (i, j) (0, otherwise), and ϵ is a small value (set to 10^{-5} in [17]).

The analysis presented in [16] for two scenarios of disasters, i.e., referring to earthquakes and use of weapons of mass destruction (WMD) shows a noticeable improvement in terms of risk reduction, however, at the expense of the increased network resource allocation. In particular, following the dependency of damage probability on the seismic hazard levels from [18], concerning the scenario of earthquakes, the model from [16] results in a reduction of risk up to 6.5% compared to the case of a conventional 1+1 protection scheme, while the ratio of resource allocation is increased by 16.5%. When analyzing the WMD case, the model provides, in turn, the reduction of risk up to 5.5% at the expense of increasing the ratio of resource allocation by 20%.

The optimization problem investigated in this section is \mathcal{NP} -complete. It is thus intractable for large networks, and the use of heuristic schemes is often the only possibility. In particular, as proposed in [17] in such a heuristic scheme, paths for connections could be established sequentially for a list of connections $t \in T$ sorted descending their penalty costs c_t using any algorithm of shortest path calculation. Following [17], the cost of link c_{ij} for working paths calculations would be:

- ∞ , if there is no more free capacity on a given link,
- ϵ , if there is enough free capacity on a given link and there is no risk of affecting a particular link by any disaster,
- defined as given in formula (4) for a non-zero probability of a failure of link (i, j) due to a disaster.

$$c_{ij} = a_{ij} + \epsilon(W_{ij} - F_{ij}) \quad \text{otherwise} \quad (4)$$

where factor a_{ij} in $(0, \infty)$ range corresponds to the respective probability in $(0, 1)$ range of a disaster-related outage of link (i, j) defined as given in formula (5), while second part of

formula (4) refers to the amount of capacity already allocated to other established paths.

$$a_{ij} = -c_t / \log(\max_{n \in N} p_{ij}) \quad (5)$$

where p_{ij} is the probability of link (i, j) failure after any outage.

For each connection, backup paths are established immediately after the corresponding working paths. For each link, its cost for backup path calculations is set to ∞ if the link belongs to the related working path. The value of ϵ is, in turn, assigned to a link not traversed by the corresponding working path and not being in any disaster zone [17].

3 Schemes of Risk-aware Re provisioning of Connections

Re provisioning operation by definition refers to rearrangement of connections following the change of the network state due to, e.g., the arrival/termination of connections, as well as a failure/repair of network elements [17, 19]. In particular, in a post-disaster scenario, re provisioning can be utilized to route each affected connection around the damaged area provided that sufficient network resources are available, and neither of the end nodes of a connection failed.

The problem to re provision connections with unaffected end nodes can be solved by applying the same scheme as given above by formulas (2)-(3) assuming that the set of connections now includes only those whose end nodes survived a disaster.

Network Preparedness against Earthquakes

A particular context of connection re provisioning refers to such operations performed in advance prior to the incoming disaster (e.g., an earthquake) feasible by utilization of earthquake early warning systems (EEWSs) so far often used by various critical infrastructure systems [20]. Such systems can provide alarms from a few seconds to a minute before the actual quake (depending on the distance). Concerning communication networks, such an interval, especially in the case of high-speed optical networks, seems sufficient to apply re provisioning of connections before they become affected by an earthquake.

Following [20], EEW systems typically consist of Earthquake Monitoring Stations (EMSs) issuing signals next processed by earthquake study centres (ESCs) which finally decide on a given alarm level. In optical networks, ECSs functionality can be given to the path

computation element (PCE), while information from EMSs can be transmitted to a PCE via optical channels.

Depending on the alarm levels, as presented in [20], it seems relevant to propose three different strategies of connection reprovisioning prior to an earthquake for:

- all connections at risk (not proper in the case of a low alarm level)
- only connections requiring the highest availability (e.g., at least five-nines of availability) – most relevant for low-risk scenarios,
- connections classified to be reprovisioned based on the actual alarm level.

Naturally, reprovisioning of end-to-end paths is feasible in each variant listed above only if both end nodes of a connection are not predicted to become affected by an earthquake.

4 Geodiverse Routing in Optical Networks

Geodiverse routing is a strategy to enhance robustness in telecommunication networks against natural disasters. Since these disasters are usually confined to a region, geodiverse routing offers geographically diverse routes to circumvent the affected region.

As already mentioned in Sect. 1, a pair of paths with the same source and destination are said to be D -geodiverse, if the geographical distance between any intermediate element (node or link) of one path is at least D from any element of the other path. This concept is explained in detail in Sect. 4.2. In [7] an approach for determining the minimum cost pair of paths which are D -geodiverse was proposed. Then, and in order to jointly satisfy availability and D -geodiverse requirements, an algorithm, denoted Guaranteed Available Pair of Geodiverse Paths (GAPGP) was presented in [21,22]. Both these algorithms are briefly described below, after the introduction of the necessary notation.

4.1 Notation

The network is represented by an undirected graph $G = (N, E)$ where N is the set of nodes and E is the set of edges representing node pairs connected by a direct link. Each edge $e \in E$, is characterized by an availability $a_e \in]0, 1]$ and a cost $c_e > 0$. We assume the network to be bi-connected, that is, no single node removal will make two nodes mutually unreachable.

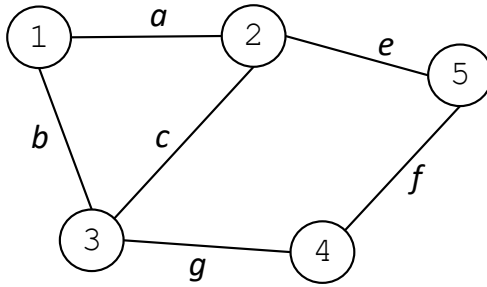


Figure 1: Network with five nodes and six links

Sometimes no path pair between nodes s and d ($s, d \in N$) satisfies the desired distance D , because the maximum achievable distance D_{sd}^{Max} is less than D . For that reason we define $D_{sd} = \min(D, D_{sd}^{Max})$ for each node pair, so the relaxed problem of finding a D_{sd} -geodiverse path pair is always feasible in bi-connected networks.

Let K designate a set of node pairs of interest; the set of all node-disjoint path pairs between a given node pair s and d in G is represented by R_{sd} and each path pair $r \in R_{sd}$ is made of two sets of edges: S_{r_1} and S_{r_2} , the edge set of the first and second path of r , respectively.

4.2 Minimum-cost Geodiverse Routing

In minimum-cost geodiverse routing, each demand from s to d is supported by a pair of D -geodiverse paths, for a given minimum D value. Among all possible D -geodiverse path pairs, the one with minimum cost is chosen.

The minimum geographical distance value D is imposed between any intermediate element of one path (excluding the end nodes and corresponding incident links) and any element of the other path (including the end nodes and corresponding incident links). This property allows to enhance network robustness to disaster based failures with a coverage diameter less than D , since if one of the paths (of the pair) is affected by the disaster in at least one of its intermediate elements (nodes and/or links), the property guarantees that the other path is not affected by the failure.

To better understand the D -geodiverse property, consider the illustrative example in Fig. 1. This illustrative network has five nodes (numbered from 1 to 5) and six links (from a to f).

Consider the source to be node 1 and the destination to be node 5. Then there is only one pair of node disjoint paths, one path defined by links $p_1 : \{a, e\}$ and the other path

defined by links $p_2 : \{b, f, g\}$. If any path uses link c , then node 2 or 3 is common implying that the paths will not be node-disjoint. Moreover, p_1 and p_2 are D -geodiverse if:

- node 2 which is the intermediate node of path p_1 is distanced at least D from nodes 1, 3, 4, 5 and from links b, f, g which are the elements of path p_2 (p_1 does not have intermediate links);
- nodes 3 and 4 which are the intermediate nodes of path p_2 are distanced at least D from nodes 1, 2, 5 and from links a, e which are the elements of path p_1 ;
- link g which is the intermediate link of path p_2 is distanced at least D from the nodes and links of p_1 .

The geographical distance between two links $e_1, e_2 \in E$ is defined as follows. If the two links have a common end node or intersect each other, then obviously $\delta(e_1, e_2) = 0$. Otherwise, $\delta(e_1, e_2)$ is given by the infimum of the geographical distances between any point in e_1 and any point in e_2 (this definition includes the cases where the links intersect each other or have a common end node). To compute the minimum-cost pair of geodiverse paths for a given distance D , a given source s and a given destination d , the geographical distance between the path links suffices.

To better understand this, consider Fig. 2 for a detailed illustration of the geographical distance between links. We can see three cases: (i) the geographical distance between links a and b is in fact the geographical distances between their closest end nodes, 2 and 3; (ii) the geographical distance between links b and c corresponds to the geographical distance of one of their inner points; (iii) and the geographical distance between links a and c is between an inner point of link c and the closest end node of link a which is 2.

The minimum-cost pair of D -geodiverse paths problem can be defined as an optimisation problem using integer linear programming (ILP), aiming to find the pair of D -geodiverse routing paths from s to d with minimum cost (for more details, refer to Sect. 3 of [7]). In Fig. 3, we can see the minimum-cost pair of D -geodiverse paths for a certain network, obtained by the ILP optimisation problem, for $D = 80$ km (left) and for $D = 120$ km (right). The black dots represent the source s and destination d nodes. For $D = 80$ km the minimum cost of a path pair is 955, while for $D = 120$ km the minimum cost of a path pair increases to 1416. Note that for larger values of D , the pair of geodiverse routing paths offers greater protection coverage against disasters, at the expense of higher costs.

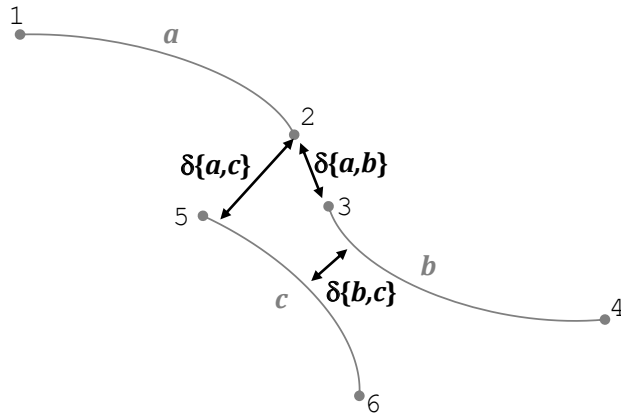
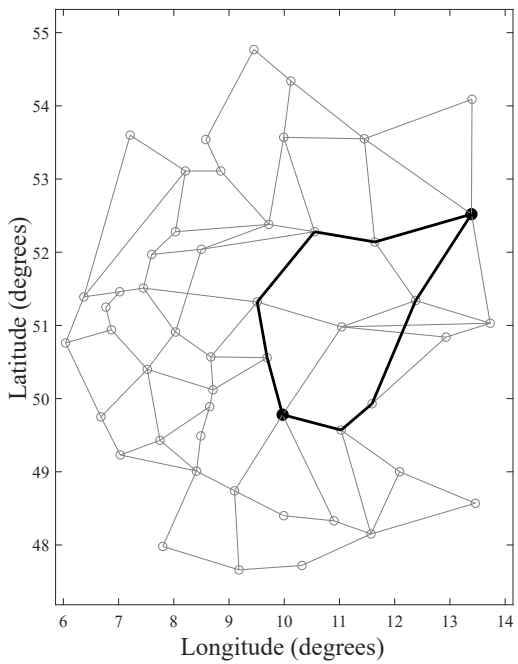
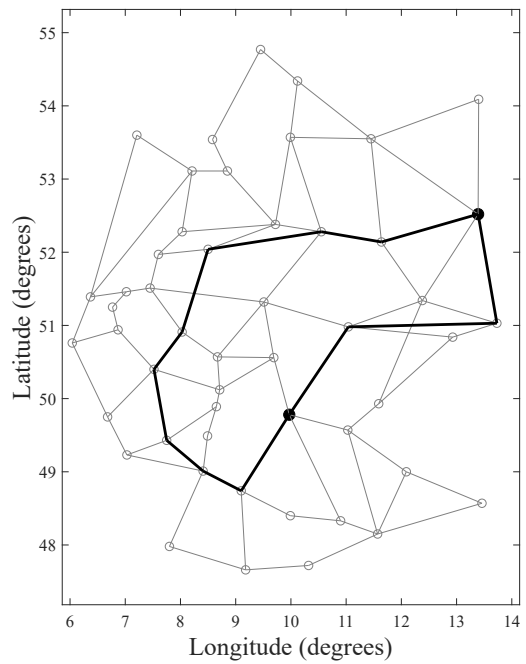


Figure 2: Geographical distance between links without common nodes



(a)



(b)

Figure 3: Minimum-cost pair of D -geodiverse paths for (a) $D = 80$ km and (b) $D = 120$ km; the black dots represent source and destination nodes

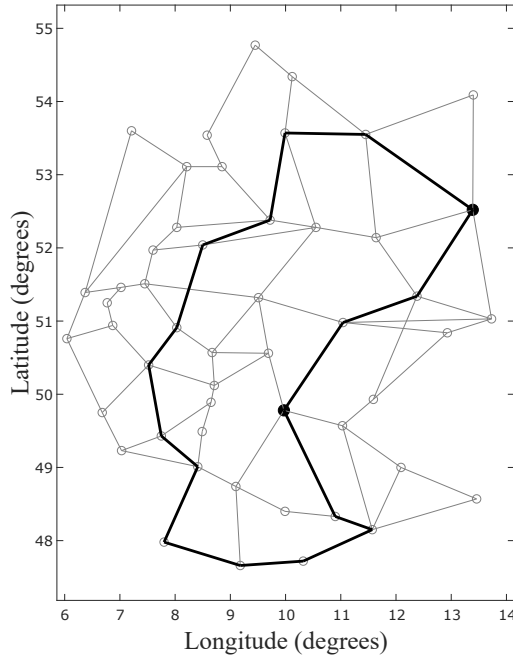


Figure 4: Minimum-cost pair of D_{sd}^{Max} -geodiverse paths; the black dots represent source and destination nodes

However, for large values of D , the problem can be infeasible. Therefore, a closely related problem is the determination of the maximum distance D_{sd}^{Max} for geodiverse routing, i.e., the maximum $D = D_{sd}^{Max}$ value for which a pair of D -geodiverse paths is still possible. This problem can also be modelled as an ILP optimisation problem (for more details, refer to Sect. 4 of [7]). In Fig. 4, for the same network and same source and destination nodes as in Fig. 3, the minimum-cost pair of D_{sd}^{Max} -geodiverse paths is shown. In this example, $D_{sd}^{Max} = 142$ km and the minimum cost is 1922.

If we consider that the network has a vulnerable region(s) (regions more susceptible to disasters), then the D -geodiverse property only needs to be ensured for the nodes and links in that region(s) (details in Sect. 5 of [7]). Vulnerable regions are usually identified as being more prone to natural disasters (earthquakes, hurricanes, etc.). In these cases the path pair only needs to be node-disjoint outside the vulnerable region and must be D -geodiverse inside the vulnerable region. If more than one vulnerable regions exist, the D -geodiversity of the path pair is only imposed between elements belonging to the same vulnerable regions.

Consider Fig. 5, where an ellipsoidal vulnerable region is highlighted in light grey. In

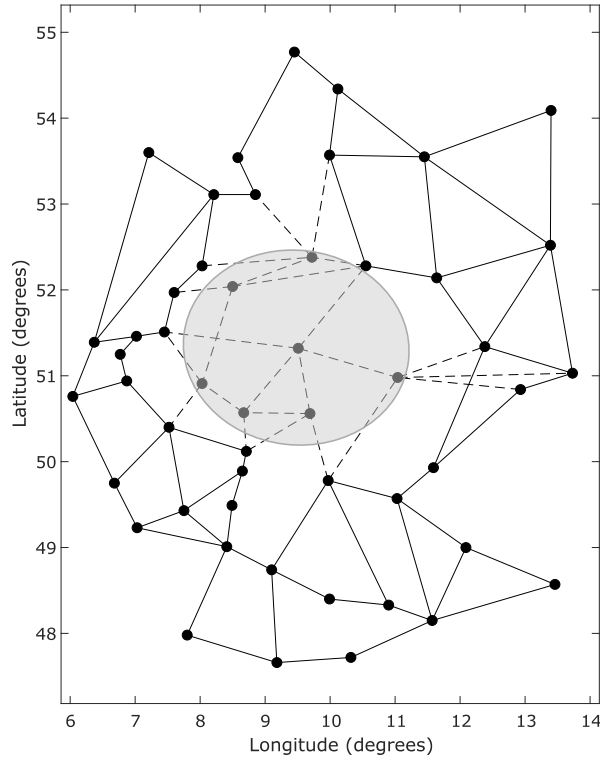


Figure 5: Highlighted vulnerable region in light grey; the nodes belonging to the vulnerable region are in grey, whereas the links belonging to the region are dashed.

Fig. 6, we can see the minimum-cost pair of D -geodiverse paths, considering the vulnerable region in Fig. 5, for $D = 100$ km (left) and for $D = 200$ km (right). The black dots represent the source and destination nodes. The dashed lines represent the links that traverse the vulnerable region. The thick grey lines represent the links of the geodiverse pair that traverses the vulnerable region (the region is not shadowed so that the links can be visible). For $D = 100$ km the minimum cost of a path pair is 1400, while for $D = 200$ km the minimum cost of a path pair increases to 1448. Note that if $D = 100$ km then we intend to protect the path pair for a predicted hazard of 100 km, and so the pair of paths of minimum cost both transverse the vulnerable region. When we consider a protection coverage of 200 km, then the minimum-cost path pair returns one of the paths circumventing the vulnerable region.

So for short-term protection against a predicted event with a known maximum coverage D , the network operator can impose D -geodiversity inside the vulnerable region. Whereas for long-term protection against eventual hazards (requiring a higher cost), the network operator can impose geodiversity with a coverage large enough to ensure that the one of

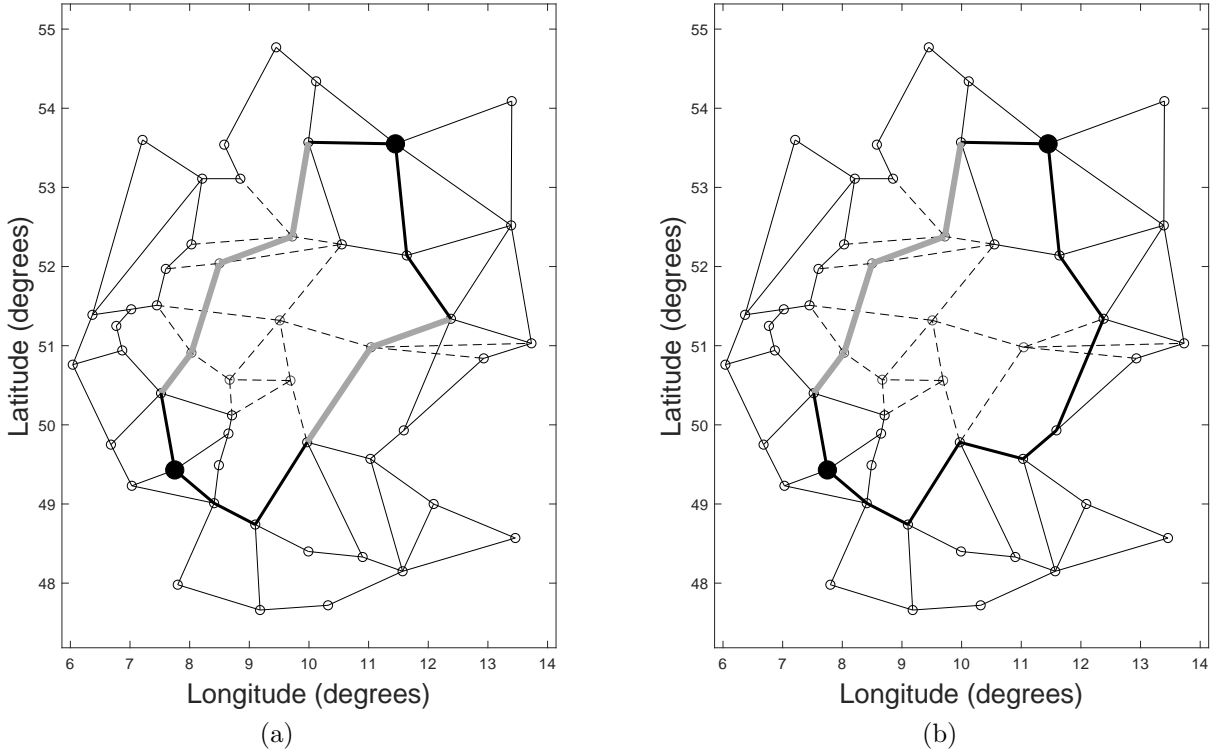


Figure 6: Minimum-cost pair of D -geodiverse paths for (a) $D = 100$ km and (b) $D = 200$ km, considering the vulnerable region; the black dots represent source and destination nodes

the paths circumvents the vulnerable region.

The ILP models, for the described problems, were solved using the CPLEX solver. Computational results show that the ILP optimisation problems can be solved efficiently for reasonably large networks (tests were done with the largest network being 75 nodes) and showed that for larger values of D , protection could be enhanced for larger coverage with greater expense costs. The results show that the increase in expense cost is smaller for networks with smaller average node degree. The results also show that the cost increase is not strongly related to the link costs.

4.3 Geodiverse Routing with Availability Constraints

Service Level Agreements (SLAs) may have multiple parameters, including a desired level of availability. As mentioned at the beginning of Sect. 4 using a pair of D -geodiverse paths may enhance network robustness against natural disasters. However it usually results in longer paths, which tend to be less reliable, as the probability of cable cuts are related

to cable lengths. Hence it is also important to be able to obtain path pairs with the desired level of availability. Moreover, if the desired level can not be achieved without some network upgrade, knowing what is the maximum availability that can be achieved under D -geodiversity constraints is an information that is relevant in the context of SLAs, or for deciding on network upgrade procedures to improve end-to-end availability.

Algorithm Guaranteed Available Pair of Geodiverse Paths (GAPGP) allows to obtain a pair of D -geodiverse paths, from a source to a destination, such that either the pair satisfies a required availability or its availability is maximal. The resolution approach used in GAPGP (explained next) is based on the enumeration of the k most available paths, and is a modification of a procedure proposed in [23].

The main idea of GAPGP is to enumerate paths by non-increasing order of their availability value. Then for each path p it calculates q , the most available path that is D -geodiverse with p . The algorithm stores the best solution found so far until no better solution can be found. The algorithm can stop enumerating paths when it generates a path p with an availability such that if a path D -geodiverse with p existed with the same availability as p , the resulting path pair would still have lower availability than the current best solution. Hence no more improvement is possible – this is similar to the condition used in [23]. Note that if path q , the D -geodiverse with p , has an availability higher than p then the corresponding path pair can be ignored: this solution will be at most as good as the solution obtained when q was enumerated before p or it has lower availability than that solution.

The availability of a path pair $r \in R_{sd}$, represented by A_r , is given by:

$$A_r = 1 - \underbrace{\left(1 - \prod_{e \in S_{r1}} a_e\right)}_{\text{unavailability of } r1} \underbrace{\left(1 - \prod_{e \in S_{r2}} a_e\right)}_{\text{unavailability of } r2}, \quad (6)$$

which is the usual formula for a parallel of serial systems.

Given a network topology, source s and destination d nodes, the link to link distances, the node to link distances (which allow to obtain $\delta(e_i, e_j)$ for a given node pair), the availability of the links, a desired distance D and availability Λ , the Algorithm GAPGP calculates a D -geodiverse path pair of availability A_r such that:

$$A_r \geq \min \left\{ \Lambda, \max_{\rho \in R_{sd}} A_\rho \right\} \quad (7)$$

Enumerating paths by non-increasing order of their availability value, can be done using a k -shortest path algorithm like Yen's [24] or the loopless version of MPS [25]. This requires the transformation of the multiplicative metric that allows to obtain the availability of path p :

$$Av(p) = \prod_{e \in p} a_e, \quad (8)$$

into an additive one:

$$c'(p) = -\ln Av(p) = -\sum_{e \in p} \ln a_e \quad (9)$$

Hence the k -th most available path is the k -th shortest path (p_k) obtained using the link cost $c'_e = -\ln a_e$, and $Av(p_k) = e^{-c'(p_k)}$, with $k = 1, 2, \dots$

Algorithm 1 Algorithm Guaranteed Available Pair of Geodiverse Paths (GAPGP)

Require: $G, s, d, \Lambda, D_{sd}, (a_e) : \forall e \in E, \delta(e_i, e_j) : \forall e_i, e_j \in E$

Ensure: $(r, \Lambda_r) \triangleright D$ -geodiverse path pair r with availability Λ_r

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1: for all  $e \in E$  do
2:    $c'_e = -\ln(a_e) \triangleright$  New edge cost for enumerating most available paths (the shortest)
3: end for
4:  $r \leftarrow (\emptyset, \emptyset); \triangleright$  Current solution  $r$ : initially there is no stored solution
5:  $\Lambda_r \leftarrow 0; \triangleright$  Lowest availability value for  $\Lambda_r$ : initially there is no stored solution
6:  $opt \leftarrow false \triangleright$  The algorithm will end when  $opt$  becomes true, i.e., when  $r$  is optimal
7: repeat
8:    $p \leftarrow \text{next-shortest-path}(s, t, G, c') \triangleright$  Next most available path
9:   if  $p$  exists then
10:    if  $Av(p)[2 - Av(p)] \leq \Lambda_r$  then  $\triangleright (p, q) : Av(p) \triangleq Av(q)$ 
11:       $opt \leftarrow true \triangleright$  The stored solution  $r$  is the optimal solution
12:    end if
13:    if  $\neg opt$  then  $\triangleright$  The optimal stopping condition was not verified
14:       $q \leftarrow \text{path-geo-distance}(\delta, D_{sd}, p, G, c') \triangleright$  Shortest path,  $D$ -geodiverse with  $p$ 
15:      if  $q$  exists then
16:        if new solution  $(p, q)$  has greater availability than the stored solution  $r$ 
17:        then
18:           $r \leftarrow (p, q) \triangleright r$  is updated with the new best found solution
19:        end if
20:      end if
21:    end if
22:  else
23:     $opt \leftarrow true \triangleright$  No more improvement possible, hence  $r$  is the optimal solution
24:  end if
25: until  $\Lambda \leq \Lambda_r \vee opt$ 

```

In the remaining part of this sub-section a detailed description of Alg. 1 (GAPGP) is given. The additive metric requires the calculation of edge cost c'_e for all $e \in E$ (lines 1–3). An empty solution is created in lines 4 and 5, which will be used to store the current best solution; in the next line *opt* – the Boolean variable that will take the value *true* once the optimal stopping condition is reached – is initially set to *false*. Then the main loop, defined by lines 7–24, is executed until the stored path attains the desired availability ($\Lambda \leq \Lambda_r$) or *opt* becomes true, meaning that the stored solution is the most available one that can be obtained, satisfying the required geodiversity constraints. The stored path pair (r with availability Λ_r) is optimal – see line 11 of GAPGP – when the availability of the current path p from s to d (obtained by non-increasing availability order) is such that $Av(p)[2 - Av(p)] \leq \Lambda_r$ (see [21,22] for a proof). Note that an explanation of this optimal condition was given in the third paragraph of this sub-section, while explaining the main idea of GAPGP.

The iterative generation of shortest paths which corresponds, as explained above, to the iterative generation of paths by non-increasing order of availability is carried out in the main cycle. In each iteration, the next most available path p is returned by function *next-shortest-path* in line 8. Path q , the D -geodiverse path with p is calculated in line 14 by function *path-geo-distance* which uses a shortest path algorithm in the auxiliary graph resulting from pruning from G the edges at a distance less than D_{sd} from p (recall $D_{sd} = \min(D, D_{sd}^{Max})$). If q could be calculated in the pruned network (see line 15), then if the resulting path pair has an availability higher than the availability of the existing stored solution, the stored solution is updated with the value of the newly found best path pair, as can be seen in lines 16–18.

To conclude, the value of variable *opt* is modified in two situations only: (a) in line 11 after having verified the current best solution has maximal availability; (b) in line 22 after function *next-shortest-path* (in line 8) returns no path because all paths have been generated. Therefore, the algorithm returns a path pair with an availability at least Λ or with the maximal availability in the network, such that the paths are D -geodiverse, as stated in Eq. (7).

4.4 On the Complexity of Finding D -Geodiverse Paths

In the literature, there are multiple related flavours of definitions of D -geodiversity. In this subsection we would like to emphasize two of them, which represent the two fundamentally different definition classes. While in one version (Def. 1, DNP, [9]), the distances between

the paths are measured only between the intermediate nodes, in the other version (Def. 2, DLP, [6, 15]), the distance between links is also measured.

Definition 1 (D-node-geodiverse path pair problem (DNP)) *Given a network $G(V, E)$ embedded in the plane, source and destination nodes s and d , respectively, decide whether a pair of paths p_1 and p_2 between s and d exists such that no node of p_1 is closer to a node of p_2 than D .*

Definition 2 (D-link-geodiverse path pair problem (DLP)) *Given a network $G(V, E)$ drawn in the plane, source and destination nodes s and d , respectively, two protective disks c_s and c_d with radius $\mathcal{R} \geq D/2$ around s and d , respectively, decide whether a pair of paths p_1 and p_2 between s and d exists such that no part (of node or link) of p_1 is closer to p_2 than D , where we neglect the parts of the paths which are inside c_s or c_d .*

While the DNP is \mathcal{NP} -hard [9, Theorem 3], the DLP is solvable in polynomial time [15, Theorem 5]. In the rest of this subsection, we will assume D -geodiversity is defined as in Def. 2. In fact, [15] and [6] offer much stronger algorithms, which, given G , D and \mathcal{R} , find a maximum cardinality set of s - d D -geodiverse paths in polynomial time¹. Regarding networks with $x \neq 0$ link crossings, FPT (fixed-parameter tractable) algorithms can be designed based on [6]. Another consequence of [6] is that finding a pair of s - d D -geodiverse paths (or $k \geq 2$ mutually D -geodiverse paths) for which the minimal bandwidth of the paths is maximal can also be done in polynomial time. The computational complexity of the cost minimization version of the DLP (i.e., finding a minimum-cost D -geodiverse path pair) is an open question. However, if $D = 0$ (or it is small enough), the problem translates to finding a minimal cost node-disjoint s - d path pair, which is solvable in polynomial time with Suurballe’s algorithm [26].

4.5 SRLG-disjoint and Geodiverse Routing – a Trade-off Between Benefit and Practical Effort

The basic path protection approach is to obtain a pair of edge disjoint paths of total minimal path length (or cost) – also designated as the link-disjoint min-sum problem. However, to ensure a better end-to-end path protection, the AP and the BP should not

¹Note that the number of link crossings is $x \ll |V|$ for backbone networks, typical networks containing (almost) no link crossings.

The maximum $D = D_{sd}^{Max}$ value for which a pair of D -geodiverse paths exists can be calculated via a binary search using the algorithm of [6].

share a common risk of failure. If two or more fibres share a cable or a duct, we say they are in a Shared Risk Link Group (SRLG) – an example can be found in Chapter 3.0/17. As the links in an SRLG share a common risk, we want path pairs to be SRLG-disjoint, i.e., they should not have any SRLG in common. Since calculating SRLG-disjoint paths is a \mathcal{NP} -complete problem [27], heuristics [28–30] are often used to determine SRLG-disjoint paths, seeking the minimization of an additive cost metric related to the links of the paths.

In this sub-section maximally SRLG-disjoint path pairs of total minimal path length are calculated. A path pair is said to be maximally SRLG-disjoint if the number of common SRLGs is minimal – see [31] for effective heuristics for solving this problem. A path pair is (fully) SRLG-disjoint if the two paths do not have any SRLG in common. Note that maximal SRLG-disjoint routing and/or D -geodiverse routing result in longer paths than routing using simply link-disjoint paths. These longer paths may require a larger number of transponders and/or more sophisticated transponders to satisfy the demands, thus increasing the optical network cost.

As SRLGs may include links in a certain vicinity sharing a common risk, they can naturally represent geographically correlated failures [32–34]. The problem addressed in this section is to determine if SRLG-disjointness already ensures a good geodiversity for a pair of paths.

For some networks, it may not be possible to find fully SRLG-disjoint path pairs for all the demands. Therefore, the problem to be tackled is based on finding maximally SRLG-disjoint path pairs of min-sum cost for all traffic demands, that will simultaneously guarantee a good geodiversity.

The minimization of the cost of the used transponders is a goal of the tackled problems. As the longest path of the pair is the relevant one for the selection of transponders, the problems were formulated so as to minimize the length of the longest path of the pair (a min-max approach), while guaranteeing that the total path pair length is the one obtained for the original min-sum models. This way, the formulated problems are of a lexicographic nature, in that ancillary problems of min-sum length are first solved, followed by the min-max length problems.

For networks with long paths, there may be demands for which fully transparent paths may not be available. In this situation, regenerative transponders (or simply transponders) will have to be used along the paths. The problem of devising the appropriate number and location of regenerators is tackled, based on an approach in [35]. This information allows us to find the longest transparent segment in the paths of each path pair, which will be

necessary to select the appropriate transponders. For simplification, we have considered identical transponders at the source, intermediate and destination nodes for each demand.

Next, a brief description of the addressed Linear Programming (LP) problems is presented – for the description of notation and a mathematical formulation, see [36]. Note that each LP problem was solved for a demand from a node s to a node d in the network:

P_0^η edge-disjoint path pair of min-sum length, with minimal length for the longest path of the pair;

P_{2N}^η node-disjoint path pair with minimal number of SRLGs in common and min-sum length, with minimal length for the longest path of the pair – identified as `min#SRLG` in the forthcoming figures;

$P_{2G}^\eta(D)$ path pair with minimal number of SRLGs in common and min-sum length, with minimal length for the longest path of the pair and geodiversity constraints, with a distance D – identified as `D=x km` in the forthcoming figures.

The resolution of these problems gives us the appropriate path pairs and also the appropriate number and location of regenerators, for each demand.

The solutions to these problems allow for an evaluation of the increasing total path length when seeking to ensure maximally SRLG-disjoint routing in a network. An assessment on whether SRLG-disjointness is enough to provide a required geodiversity is also relevant, which results in problem $P_{2G}^\eta(D)$ listed above. These LP problems were solved using CPLEX 12.8 [37]. Recall that if for a particular demand $s - d$ a distance D cannot be satisfied then the maximum possible geodiverse solution is sought, i.e., $D_{sd} = \min(D; D_{sd}^{Max})$ where D_{sd}^{Max} is calculated as in Sect. 4.1 with an additional constraint on the minimal number of shared SRLGs for that demand (which has to be calculated in advance).

A sample backbone network of Deutsche Telekom (DT) with 12 nodes, see Fig. 7, was considered in the experimental analysis. Information on the fibre lengths, the SRLGs, and the SRLG lengths may be found at Tables II-III of [38]. The distance between links, the distance between the nodes and the links, and the maximal distance between paths for every node pair, can be found at [39]. The considered distances were $D = 40; 80; 120; 160$ km. Physical constraints regarding optical reach were not taken into account at this stage.

The length variation of the paths in relation to the length of the paths for the P_0^η solution, is displayed in Fig. 8.

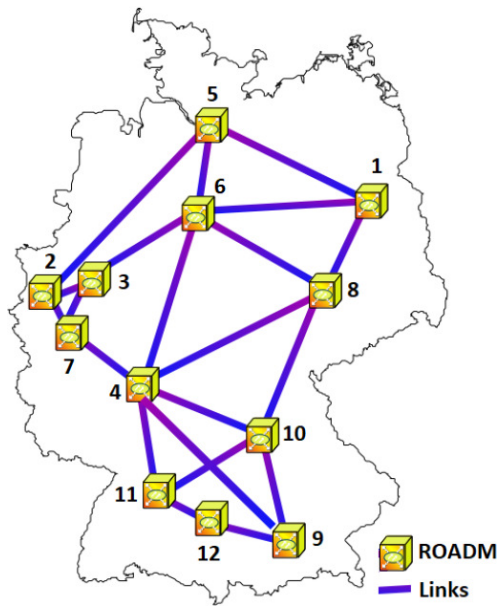


Figure 7: Generic backbone network of Deutsche Telekom

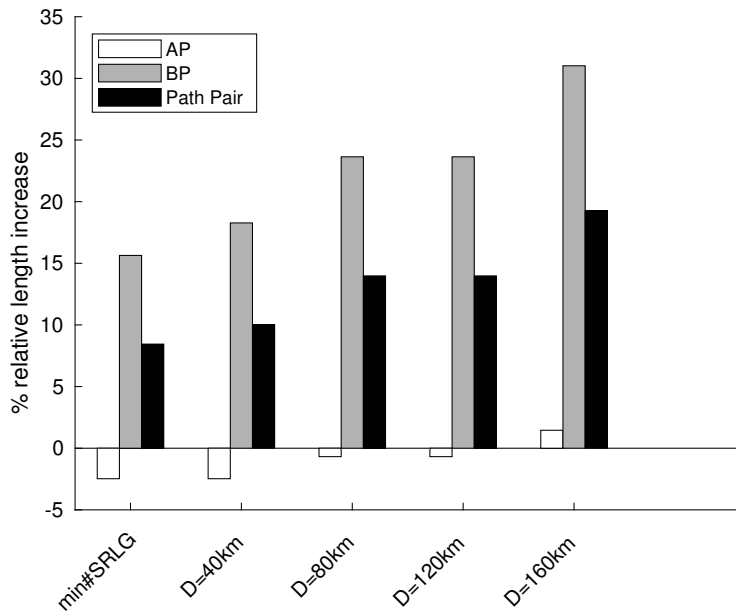


Figure 8: Path length variation (to the solutions of P_0^η)

For the considered network, only for one node (2) it is not possible to reach any other node with a fully SRLG-disjoint path pair. For demands with this node as source or destination, one of two possible SRLGs has to be shared.

For the APs, the solutions have smaller variations for all the problems. For the demands (2,12), (3,12) and (7,12), the APs for all the problems are substantially shorter than the APs for P_o^η , originating negative variations of the length of the APs. This has to do with the topology of the network and with the set of SRLGs that was considered. For the BPs and for the path pairs, the solutions are more diversified.

Generally speaking, the results support our initial conjecture that SRLG-disjointness ensures a certain geodiversity. In fact, the increase in total path length from $\text{min}\#\text{SRLG}$ to the geodiverse solutions oscillates only between 1.5% for $D=40\text{km}$ and 10.0% for $D=160\text{km}$, i.e., to achieve a geodiversity of $D = 160 \text{ km}$, the length of the path pairs increases only 10.0% in relation to the path pairs length obtained for $\text{min}\#\text{SRLG}$. The increase in the length for the path pairs of the solutions considering geodiversity varies from 10.0% (for $D=40\text{km}$) to 19.3% (for $D=160\text{km}$), compared to the P_o^η solution. Naturally, for higher D , the BPs are longer, to ensure that the desired geodiversity is achieved.

A final note on these results, to mention that in this network the results for $D=80\text{km}$ and $D=120\text{km}$ are the same.

For the demands with distant source and destination, the difference between the P_o^η solution and the solutions with SRLG-disjointness with geodiversity are not very significant. The increase in total path length is essentially due to the demands with close source and destination.

Considering the solutions obtained for the different problems and a traffic matrix, a cost may be calculated for each solution. This cost is associated with the reconfigurable optical add-drop multiplexers (ROADMs) that must be used. For this purpose, the fibre length of the paths is considered, in order to calculate the optical reach and the maximum rate of the used transponders. Note that the capacity of the links is assumed to be enough to accommodate all the traffic demands.

The optical reach of a lightpath is given by $N_{s_{\max}} L_s$, where L_s is the length of the amplifier spans and $N_{s_{\max}}$ is the maximum number of spans for a minimum Optical Signal-to-Noise Ratio (OSNR). The calculation of these parameters is based on the model described in [40]. The values of the different parameters needed for the calculation of the transponders reach are provided in [36], except the attenuation constant, which is $\alpha = 0.25 \text{ dB/km}$ (a more realistic value), and the OSNR penalty value, which is 2.2 dB.

The cost of the solutions is based on the costs of the used transponders. A state-of-the-art 100 Gb/s, Dual-Polarisation Quadrature Phase-Shift Keying (DP-QPSK) transponder with a single carrier and a baud-rate of 34 Gbaud is the normalized transponder (i.e., with cost 1). The cost of other transponders is calculated in relation to this normalized cost and depends on three main factors: (i) the number of optical carriers (lasers) – a transponder with two or more carriers is more cost-efficient than two or more transponders with a single carrier; (ii) the maximum baud-rate – an increase of the maximal baud-rate to its double entails an increase of about one-third in the cost; (iii) the adaptability of the transponder capacity – a fixed transponder supporting a single modulation format and thus a single capacity is cheaper than a modulation-flexible transponder supporting multiple modulation formats and related capacities. The effects of these factors in the cost of transponders is illustrated in Table I in [38]. The least cost transponder is always selected and the traffic is split if more than one transponder is necessary.

As already mentioned, the resolution of the aforementioned problems P_o^η , P_{2N}^η and $P_{2G}^\eta(\mathbf{D})$ allows us to find not only the appropriate path pairs but also the appropriate number and location of regenerators, for each demand. Given this information, the set of transponders with minimal cost to be used for each demand is obtained as described in Sect. 4.6 of [36]. The resolution approach is lexicographic, in the sense that once we obtain the minimal number of intermediate nodes where regenerators will be necessary, we seek to minimize the longest transparent segment in any of the paths of the pair for each demand. Any transponder with a reach higher than the maximal length of a transparent segment in a path may be used, so by minimizing this maximal length, we manage to diversify the usable transponders.

Among the possible transponders, the ones with lower cost are selected. As a simplification, the same type of transponders is used at the source, intermediate and destination nodes for each demand, regardless of the actual length of each individual segment.

The cost variation of the transponders in relation to the transponders costs for the P_o^η solution, is displayed in Fig. 9. The traffic matrix can be found in Table IV of [38].

As explained, the length of the paths (in particular of the BPs, which are usually longer than the APs) influences the cost of the transponders (used by both paths in the pair). In fact, the results in Fig. 9 show that the solutions with longer paths have equal or higher transponders costs. Note, however, that the increase is not proportional: by comparison to the P_o^η solution, the increase in BP length varies between 15.6% and 31.0%, and yet the increase in transponders costs varies between 7.1% and 8.0% only. Note that only a

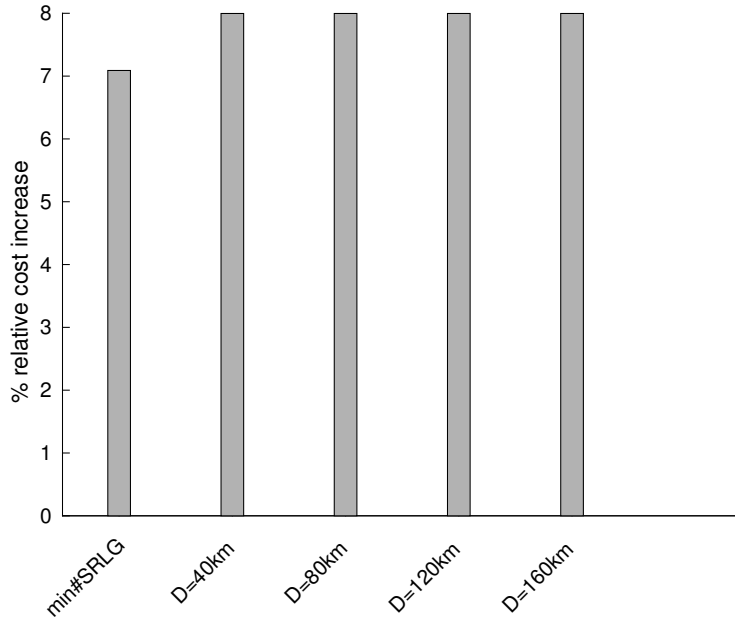


Figure 9: Transponders cost variation (to the solutions of P_0^n)

few demands (6 for the SRLG-disjoint solutions and 7 for the geodiverse solutions) require higher cost transponders than the ones needed by the P_0^n solution.

Nonetheless, the obtained results show that the length of the paths is not the sole criterion. For paths of different total length but with the same number of transponders and similar value for the length of the longest transparent segment, we may have a similar set of transponders. This is noticeable in these experimental results as the transponders costs for the geodiverse solutions are all the same. In fact, for the network in Fig. 7 and the used parameters all end-to-end connections are transparent. Experimental results for larger networks are available in [36].

5 Conclusions

Large-scale disasters such as hurricanes, tsunamis, or earthquakes may have a significant impact in the telecommunication services in a certain geographical area. This can affect communications required to coordinate the efforts of teams involved in relief operations, and can have an impact on economic activities.

Disaster-resilient routing schemes for regional failures were presented starting with an overview of routing strategies that may help circumventing damaged areas of the networks. Prior knowledge about vulnerable regions, can be used to avoid regions with high probab-

ity of disaster. Optimisation models were described that allow to establish connections with improved resistance to disasters, namely earthquakes. Moreover reprovisioning schemes, acting in response to earthquake early warning systems, were also addressed.

A detailed description of a minimum-cost geodiverse routing approach, where a pair of end-to-end paths, such that any intermediate element of one path and any element of the other path are separated by a minimum geographical distance value D was presented. Such paths are a D -geodiverse routing solution. Furthermore an algorithm, which ensures a certain level of availability is also satisfied by the obtained D -geodiverse routing solution, is described. This algorithm may allow a service provider to better specify availability levels in SLAs.

Although geodiverse routing may reduce the impact of a natural disaster, as it contributes to a higher level of disaster preparedness, it comes with a cost: longer paths which require more sophisticated terminal equipment. Hence, a case study in an optical network is presented, that evaluates the relative additional cost due to the necessary terminal equipment (transponders), needed to support a much larger separation of the selected routes, with respect to the equipment that would suffice to support minimal path length link-disjoint routes.

References

- [1] Y. Cheng, J. Li, and J. P. G. Sterbenz. Path geo-diversification: Design and analysis. In *2013 5th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, pages 46–53, Sept 2013.
- [2] F. Iqbal and F. A. Kuipers. *Wiley Encyclopedia of Electrical and Electronics Engineering*, chapter Disjoint Paths in Networks (Invited paper). John Wiley & Sons, 2015.
- [3] T. Tsubaki, M. Ishizuka, and S. Yasukawa. A new algorithm of route design against large-scale disasters. In *NOMS 2018 - 2018 IEEE/IFIP Network Operations and Management Symposium*, pages 1–5, Taipei, Taiwan, 2018.
- [4] Y. Cheng, M. T. Gardner, J. Li, R. May, D. Medhi, and J. P. G. Sterbenz. Analysing geopath diversity and improving routing performance in optical networks. *Computer Networks*, 82:50–67, 2015.

- [5] Y. Cheng, M. T. Gardner, J. Li, R. May, D. Medhi, and J. P. G. Sterbenz. Optimised heuristics for a geodiverse routing protocol. In *2014 10th International Conference on the Design of Reliable Communication Networks (DRCN)*, pages 1–9, April 2014.
- [6] K. Otsuki, Y. Kobayashi, and K. Murota. Improved max-flow min-cut algorithms in a circular disk failure model with application to a road network. *European Journal of Operational Research*, 248(2):396–403, 2016.
- [7] A. de Sousa, D. Santos, and P. Monteiro. Determination of the minimum cost pair of D -geodiverse paths. In *DRCN 2017 - 13th International Conference on the Design of Reliable Communication Networks*, Munich, Germany, March 8-10 2017.
- [8] Y. Cheng, D. Medhi, and J. P. G. Sterbenz. Geodiverse routing with path delay and skew requirement under area-based challenges. *Networks*, 66(4):335–346, 2015.
- [9] S. Trajanovski, F. A. Kuipers, A. Ilić, J. Crowcroft, and P. Van Mieghem. Finding critical regions and region-disjoint paths in a network. *IEEE/ACM Transactions on Networking*, 23(3):908–921, June 2015.
- [10] J. Wang, J. Bigham, and C. Phillips. A geographical proximity aware multi-path routing mechanism for resilient networking. *IEEE Communications Letters*, 21(7):1533–1536, July 2017.
- [11] Y. Cheng and J. P. G. Sterbenz. Critical region identification and geodiverse routing protocol under massive challenges. In *2015 7th International Workshop on Reliable Networks Design and Modeling (RNDM)*, pages 14–20, Oct 2015.
- [12] M. W. Ashraf, S. M. Idrus, and F. Iqbal. Maximally spatial-disjoint lightpaths in optical networks. *Indonesian Journal of Electrical Engineering and Computer Science*, 10(2):733–740, May 2018.
- [13] M. W. Ashraf, S. M. Idrus, F. Iqbal, and R. A. Butt. On spatially disjoint lightpaths in optical networks. *Photonic Network Communications*, 36(11):11–25, Aug. 2018.
- [14] A. Sen, S. Murthy, and S. Banerjee. Region-based connectivity – A new paradigm for design of fault-tolerant networks. In *2009 International Conference on High Performance Switching and Routing*, pages 1–7, Paris, France, June 2009.

- [15] Y. Kobayashi and K. Otsuki. Max-flow min-cut theorem and faster algorithms in a circular disk failure model. In *IEEE INFOCOM 2014 - IEEE Conference on Computer Communications*, pages 1635–1643, April 2014.
- [16] F. Dikbiyik, A. S. Reaz, M. De Leenheer, and B. Mukherjee. Minimizing the disaster risk in optical telecom networks. In *OFC/NFOEC*, pages 1–3, March 2012.
- [17] F. Dikbiyik, M. Tornatore, and B. Mukherjee. Minimizing the risk from disaster failures in optical backbone networks. *Journal of Lightwave Technology*, 32(18):3175–3183, Sep. 2014.
- [18] G. Grünthal. European macroseismic scale 1998 (ems-98), cahiers du centre europeen de geodynamique et de seismologie. *Cahiers du Centre Europeen de Geodynamique et de Seismologie*, 15, 1998.
- [19] L. Song, J. Zhang, and B. Mukherjee. A comprehensive study on backup-bandwidth reprovisioning after network-state updates in survivable telecom mesh networks. *IEEE/ACM Transactions on Networking*, 16(6):1366–1377, Dec 2008.
- [20] M. Oguz, F. Dikbiyik, and H. S. Kuyuk. Earthquake preparedness strategies for telecom backbone with integration of early warning systems and optical WDM networks. In *2016 8th International Workshop on Resilient Networks Design and Modeling (RNDM)*, pages 181–188, Sep. 2016.
- [21] A. de Sousa, T. Gomes, R. Girão-Silva, and L. Martins. Minimizing the network availability upgrade cost with geodiversity guarantees. In *2017 9th International Workshop on Resilient Networks Design and Modeling (RNDM)*, pages 1–8, Sept. 4-6 2017.
- [22] A. de Sousa, T. Gomes, R. Girão-Silva, and L. Martins. Minimization of the network availability upgrade cost with geodiverse routing for disaster resilience. *Optical Switching and Networking*, 31:127–143, Jan. 2019.
- [23] T. Gomes and J. Craveirinha. Efficient calculation of the most reliable pair of link disjoint paths in telecommunication networks. *European Journal of Operational Research*, 182(3):1055–1064, 2007.
- [24] J. Y. Yen. Finding the k shortest loopless paths in a network. *Management Science*, 17(11):712–716, July 1971.

- [25] E. Martins, M. Pascoal, and J. Santos. Deviation algorithms for ranking shortest paths. *International Journal of Foundations of Computer Science*, 10(3):247–263, 1999.
- [26] J. W. Suurballe and R. E. Tarjan. A quick method for finding shortest pairs of disjoint paths. *Networks*, 14(2):325–336, 1984.
- [27] J. Q. Hu. Diverse routing in optical mesh networks. *IEEE Transactions on Communications*, 51(3):489–494, March 2003.
- [28] T. Gomes, C. Simões, and L. Fernandes. Resilient routing in optical networks using SRLG-disjoint path pairs of min-sum cost. *Telecommunication Systems Journal*, 52(2):737–749, 2013.
- [29] P. Laborczi, J. Tapolcai, P.-H. Ho, T. Cinkler, A. Recski, and H. T. Mouftah. Algorithms for asymmetrically weighted pair of disjoint paths in survivable networks. In T. Cinkler, editor, *Proceedings of Design of Reliable Communication Networks (DRCN)*, pages 220–227, Budapest, Hungary, October 7-10 2001.
- [30] A. Todimala and B. Ramamurthy. IMSH: An iterative heuristic for SRLG diverse routing in WDM mesh networks. In *13th International Conference on Computer Communications and Networks, ICCCN'2004*, pages 199–204, October 2004.
- [31] T. Gomes, L. Jorge, P. Melo, and R. Girão-Silva. Maximally node and SRLG-disjoint path pair of min-sum cost in GMPLS networks: a lexicographic approach. *Photonic Network Communications*, 31(1):11–22, 2016.
- [32] J. Tapolcai, L. Rónyai, B. Vass, and L. Gyimóthi. List of shared risk link groups representing regional failures with limited size. In *IEEE INFOCOM 2017 - IEEE Conference on Computer Communications*, pages 1–9, May 2017.
- [33] B. Vass, E. Bérczi-Kovács, and J. Tapolcai. Enumerating shared risk link groups of circular disk failures hitting k nodes. In *DRCN 2017 - 13th International Conference on the Design of Reliable Communication Networks*, pages 1–9, Munich, Germany, March 8-10 2017.
- [34] B. Vass, L. Németh, and J. Tapolcai. The Earth is nearly flat: Precise and approximate algorithms for detecting vulnerable regions of networks in plane and on sphere. *Networks (Accepted)*, Wiley, 2019.

- [35] A. Beshir, F. Kuipers, A. Orda, and P. Van Mieghem. Survivable routing and regenerator placement in optical networks. In *IV International Congress on Ultra Modern Telecommunications and Control Systems (ICUMT 2012)*, pages 684–690, St. Petersburg, Russia, Oct. 3-5 2012.
- [36] R. Girão-Silva, B. Nedic, M. Gunkel, and T. Gomes. A study on SRLG-disjointness and geodiverse routing – a trade-off between benefit and practical effort. *Submitted to Networks*, 2018.
- [37] *IBM ILOG CPLEX Optimization Studio V12.8*. IBM, 2017.
- [38] B. Nedic, M. Gunkel, T. Gomes, and R. Girão-Silva. SRLG-disjointness and geodiverse routing – a practical network study and operational conclusions. In *10th International Workshop on Resilient Networks Design and Modeling (RNDM 2018)*, pages 1–8, Longyearbyen, Svalbard (Spitsbergen), Norway, Aug. 27-29 2018.
- [39] <http://www.deec.uc.pt/~teresa/rndm2018>, June 2018.
- [40] P. Poggiolini, A. Carena, V. Curri, G. Bosco, and F. Forghieri. Analytical modeling of nonlinear propagation in uncompensated optical transmission links. *IEEE Photonics Technology Letters*, 23(11):742–744, June 2011.